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Two-beam free-electron laser

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A model of a free electron laser (FEL) amplifier operating simultaneously with two electron beams of different energy is presented. The electron beam energies are chosen so that the fundamental resonance of the higher energy beam is at a harmonic of the lower energy beam. By seeding the lower energy FEL interaction with its fundamental radiation wavelength, an improved coherence of the unseeded higher energy FEL emission is predicted. This method may offer an important alternative to those seeding proposals for FELs currently under development in the xuv and x-ray regions of the spectrum.

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I. INTRODUCTION

High gain free electron laser (FEL) amplifiers should be able to generate intense, coherent radiation in the xuv and x ray regions of the spectrum. Such FEL systems form the main component in the drive toward the next, fourth generation, light sources. The electromagnetic radiation that will be amplified in such FEL amplifiers may arise from either the intrinsic noise of the system or from a combination of this noise and an externally injected seed field. Amplification of the intrinsic noise only has been called self-amplified spontaneous emission (SASE) [1] which is characterized by a noisy FEL output at saturation with relatively poor temporal coherence and large fluctuations [2]. The simplest conceptual method to resolve this problem is to inject a well-formed resonant coherent seed field at the beginning of the FEL interaction that dominates the intrinsic noise. The FEL output is then significantly improved over that of SASE at saturation. However, there are, as yet, no such seed sources available in the xuv and x-ray regions. Seeding at longer wavelengths can generate shorter wavelengths by using the two-wiggler harmonic method of [3]. Variations on this theme have been suggested and implemented [4]. Other methods propose using a monochromator either at the early stages of the FEL interaction [5] or with some feedback [6] to improve temporal coherence. In this paper an alternative method of seeding, based on a two-electron-beam FEL interaction, is proposed and investigated in the one-dimensional (1D) limit. This proposed application of the two-beam FEL is used to demonstrate what the authors believe may be a potentially rich regime of FEL physics.

II. THE MODEL

We propose a simple FEL wiggler system of constant period λ_w and field strength B_w through which two electron beams of different energy copropogate. The lower energy electron beam has a Lorentz factor of γ_1 and the higher energy γ_n. The higher energy electron beam is chosen so that its fundamental resonant wavelength is a harmonic resonant wavelength of the lower energy beam. Then, it may easily be shown from the FEL resonance relation $\lambda=\lambda_w(1+a_w^2)/2\gamma^2$ that $\gamma_n=\sqrt{n}\gamma_1$. It should then be possible to seed the copropagating electron beams with an externally injected seed radiation field at the fundamental of the lower energy electron beam. If such a seed field is significantly above the noise level then the lower energy electrons will begin to bunch at their fundamental resonant wavelength and retain the coherence properties of the seed. Such bunching at the fundamental also generates significant components of bunching at its harmonics which can also be expected to retain the coherence properties of the seed. In a planar FEL, this also results in radiation emission at these harmonics. This process should couple strongly with the copropagating higher energy beam whose fundamental FEL interaction is at one of the lower energy beam's harmonics. This coupling between lower and higher energy FEL interactions may allow the transferral of the coherence properties of the longer wavelength seed field to the unseeded shorter harmonic wavelength interaction.

Another coupling between the lower and higher energy electron beams, which has the potential to degrade beam quality, is the two-stream instability [7]. Using the results of [7], however, it can be shown that the instability is either below threshold or has an insignificant effect for electron beam currents ($\gtrsim$1 kA) and energies ($\gtrsim$500 MeV) typical to those used in the FEL interactions presented here.

The physics of the planar wiggler FEL in the 1D Compton limit may be described by the coupled Maxwell-Lorentz equations which, under the simplifying assumptions, universal scaling and notation of [1,8], are written

$$\frac{d\vartheta_j}{d\bar{z}}=p_j, \tag{1}$$

$$\frac{dp_j}{d\bar{z}}=-\sum_{h,odd}F_h(A_h e^{ih\vartheta_j}+\text{c.c.}), \tag{2}$$

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$$\frac{dA_h}{d\bar{z}} = F_h \langle e^{-ih\vartheta} \rangle, \tag{3}$$

where $j=1,\ldots,N$ are the total number of electrons, $h=1$, 3, 5,… are the odd harmonic components of the field, and F_h are the usual difference of Bessel function factor associated with planar wiggler FELs. This set of equations (1)–(3) is used to describe the FEL interaction of the lower energy (γ_1) electron beam.

Strongest coupling between the lower and higher energy electron beams would be expected for the lowest harmonic interaction $h=n=3$ so that the higher energy electron beam has energy $\gamma_3=\sqrt{3}\gamma_1$. Higher harmonic interactions $h=n>3$ may also be of interest, as those harmonics $h<n$ of the lower energy beam are not resonant with the higher energy beam and would not be expected to unduly disrupt the coupling to the higher harmonic.

The equations describing the FEL interaction of the higher energy electron beam, with its fundamental resonant field only, may be written in the similar form

$$\frac{d\varphi_j}{dz'} = \mathfrak{p}'_j, \tag{4}$$

$$\frac{d\mathfrak{p}'_j}{dz'} = -F_1(A'_1 e^{i\varphi_j} + \text{c.c.}), \tag{5}$$

$$\frac{dA'_1}{dz'} = F_1 \langle e^{-i\varphi} \rangle. \tag{6}$$

We shall neglect harmonics of the higher energy electron beam as these will have a significantly weaker coupling to the lower energy beam, e.g., if $n=3$ then the third harmonic of higher energy beam will be the ninth harmonic of the lower energy beam.

In their universally scaled forms, the two sets of equations (1)–(6) have different Pierce parameters [1], ρ, due to the different energy and current density of the electron beams. By using the usual relations between scaled and unscaled fields and lengths [1], Eqs. (4)–(6) may be written with the same scaling of Eqs. (1)–(3) to give the final set of working equations describing the coupled FEL system:

$$\frac{d\vartheta_j}{d\bar{z}} = p_j, \tag{7}$$

$$\frac{d\varphi_j}{d\bar{z}} = \mathfrak{p}_j, \tag{8}$$

$$\frac{dp_j}{d\bar{z}} = -\sum_{h,odd}^{n} F_h(A_h e^{ih\vartheta_j} + \text{c.c.}), \tag{9}$$

$$\frac{d\mathfrak{p}_j}{d\bar{z}} = -c_1(F_1 A_n e^{i\varphi_j} + \text{c.c.}), \tag{10}$$

$$\frac{dA_h}{d\bar{z}} = S_{h\vartheta}, \tag{11}$$

$$\frac{dA_n}{d\bar{z}} = S_\varphi + S_{n\vartheta}, \tag{12}$$

where

$$S_{k\vartheta} \equiv F_k \langle e^{-ik\vartheta} \rangle, \quad S_\varphi \equiv c_2 F_1 \langle e^{-i\varphi} \rangle, \tag{13}$$

$$c_1 = \frac{1}{n^{1/4}} \left(\frac{\rho_n}{\rho_1} \right)^{3/2} = \frac{1}{n} \sqrt{\frac{I_n}{I_1}}, \tag{14}$$

$$c_2 = n^{1/4} \left(\frac{\rho_n}{\rho_1} \right)^{3/2} = \frac{1}{\sqrt{n}} \sqrt{\frac{I_n}{I_1}}, \tag{15}$$

and in (11) h refers to all odd harmonics $h<n, I$ is the beam current, and subscripts 1 (n) refer to the parameters of the lower (higher) energy beam. Note that all harmonic interactions have been assumed negligible for $h>n$ in Eq. (9). By assuming both beams have the same transverse cross section (or equivalently the same normalized transverse emittance in a common, matched focusing channel through the wiggler) then $\rho_{1,n} \propto I_{1,n}^{1/3}/\gamma_{1,n}$ and the second equalities of (14) and (15) are obtained in terms of the beam currents. This assumption is applied to the work presented hereafter. Note that Eqs. (3) and (6), describing evolution of the fields A_n and A'_1, refer to the *same* field, which, once (6) has been rescaled, allows the two driving terms to be combined into the single differential equation (12) for the harmonic field A_n.

The coupling of the low energy electrons to both fundamental and harmonic fields is seen in Eq. (9). From Eq. (10), the higher energy electrons only couple to the harmonic field A_n (their fundamental). The fields A_h are subharmonic to the higher energy electrons and are therefore not resonant. The fields A_h are seen from Eq. (11) to be driven only by the lower energy electron beam (the higher energy electrons are not resonant with A_h). Equation (12) demonstrates that the harmonic field has two driving sources, both the lower and higher energy electron beams. From these couplings it is seen that, whereas the shorter wavelength radiation field is directly coupled to both lower and higher energy electron beams, the longer wavelength has no direct coupling with the higher energy beam. In this sense, the short wavelength harmonic interaction may be described as "parasitic," as it may resonantly extract energy directly from both lower and higher energy electron beams, whereas the longer wavelength may only directly extract energy from the lower energy beam.

The working equations readily yield the constant of motion corresponding to conservation of energy,

$$\sum_{h,odd}^{n} |A_h|^2 + \langle p \rangle + \sqrt{n} \langle \mathfrak{p} \rangle. \tag{16}$$

It is seen from the definitions of the scaled electron energy parameters $p_j \equiv (\gamma_j - \gamma_1)/\rho_1\gamma_1$ and $\mathfrak{p}_j \equiv (\gamma_j - \gamma_n)/\rho_1\gamma_n$ that the electron beam energy relation $\gamma_n = \sqrt{n}\gamma_1$ accounts for the factor of $\sqrt{n}$ in (16).

A linear analysis of the system (7)–(12) has been carried out using the method of collective variables [1]. Assuming resonant interactions for both electron beams, and that both

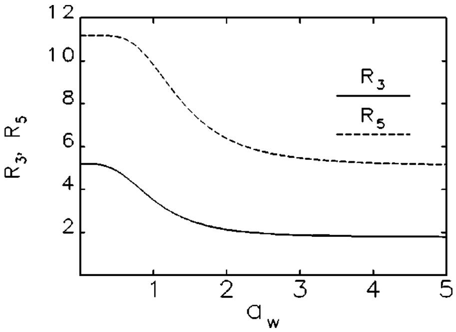

FIG. 1. The critical value of R_n, above which the shorter wavelength harmonic field has a higher gain than the longer wavelength, as a function of wiggler parameter a_w, for $n=3$, 5.

beams are effectively "cold" so that neither emittance nor energy spread have a deleterious effect upon the FEL interaction, this analysis yields a condition for the beam current ratio, $R_n \equiv I_n/I_1$, above which gain at the harmonic is greater than gain at the longer wavelength

$$R_n > n\sqrt{n}\left(1 - \frac{n|F_n|^2}{|F_1|^2}\right). \qquad (17)$$

The critical value for R_n is plotted as a function of wiggler parameter a_w for the case of two different high energy beams $n=3$ and $n=5$ in Fig. 1.

III. NUMERICAL SIMULATION

In order to demonstrate the evolution of the coupled two-beam FEL system, the working Eqs. (7)–(12) were solved numerically. A relatively simple system is considered with two initially monoenergetic and noiseless electron beams with $n=3$ and of current ratio $R_3=2$ copropagating in a wiggler of parameter $a_w=2$. It is seen from Fig. 1 that this value of R_3 is just below the threshold, so that the longer wavelength interaction will have a slightly larger growth rate in the linear regime. The seed field at the longer wavelength is modeled by defining its initial scaled intensity at the beginning of the FEL interaction $|A_1(\bar{z}=0)|^2$ to be two orders of magnitude greater than that of the harmonic. In Fig. 2 the scaled intensities $|A_1|^2$ and $|A_3|^2$ are plotted as a function of scaled distance through the FEL interaction region. Figure 3 plots the modulus of the bunching parameters $b_k \equiv \langle e^{-ik\vartheta}\rangle$ for $k=1$, 3 of the lower energy beam and $b_\varphi \equiv \langle e^{-i\varphi}\rangle$ of the higher energy electron beam. The feature of interest from Fig. 2 is the rapid growth in the harmonic intensity $|A_3|^2$ by nearly two orders of magnitude in the interval $6.5 \lesssim \bar{z} \lesssim 8$. The harmonic intensity is then further amplified by nearly another two orders of magnitude until saturation at $\bar{z} \approx 13.5$. From Fig. 3 it would appear that the period of rapid growth experienced by $|A_3|^2$ is driven by the bunching $|b_3|$ of the lower energy beam. This harmonic bunching is caused, not necessarily by electrons bunching at the harmonic wavelength, but by the significant harmonic component of the strong bunching at the fundamental $|b_1|$ as the lower energy FEL interaction approaches its saturation at $\bar{z} \approx 9$ [3,8]. This harmonic bunching, and subsequent harmonic emission of A_3 from the lower energy beam, can be expected to retain the coherence properties of the initial radiation seed field at the fundamental. This process should therefore act as a harmonic seed field with good coherence properties. Following this harmonic seeding by the lower energy electron beam it is seen that the harmonic intensity continues exponential growth until saturation.

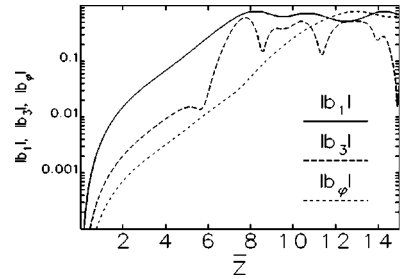

FIG. 3. The bunching parameters $|b_{1,3}|$ and $|b_\varphi|$ as a function of scaled distance $\bar{z}$ through the FEL interaction region for $n=3$.

Figure 4 plots individually the moduli of the driving terms $S_\varphi = \sqrt{R_3/3}\, F_1\, |b_\varphi|$ and $S_{3\vartheta} = F_3|b_3|$ of the harmonic field evolution equation (12). The contribution S_φ is due to the higher energy electron beam, and $S_{3\vartheta}$, the lower energy beam. The figure clearly shows the seeding phase between $6.5 \lesssim \bar{z} \lesssim 8$, as discussed above, where the harmonic field is strongly driven by the lower energy beam ($S_{3\vartheta} > S_\varphi$). This is followed by the amplification phase where the harmonic field is driven by the higher energy beam ($S_\varphi > S_{3\vartheta}$).

The above results also suggest that a hybrid high gain harmonic generation (HGHG) scheme may be possible if the interaction is stopped at $\bar{z} \approx 8.0$, and the mildly bunched

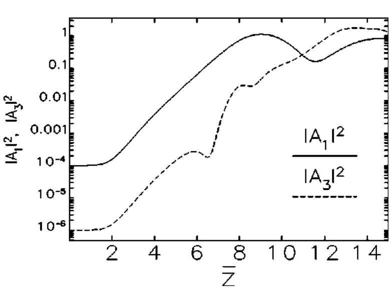

FIG. 2. The scaled radiation intensities $|A_1|^2$ and $|A_3|^2$ as a function of scaled distance $\bar{z}$ through the FEL interaction region for $n=3$.

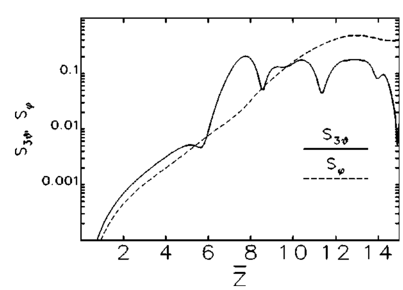

FIG. 4. The driving terms S_φ and $S_{3\vartheta}$ of the harmonic field A_3 as a function of scaled distance through the FEL interaction region for $n=3$.

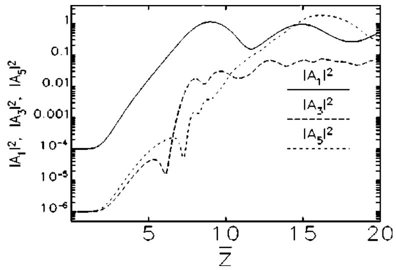

FIG. 5. The scaled radiation intensities $|A_1|^2, |A_3|^2$, and $|A_5|^2$ as a function of scaled distance $\bar{z}$ through the FEL interaction region for $n=5$.

($|b_\varphi| \approx 5 \times 10^{-2}$) higher energy electron beam is injected into another wiggler, which would allow for emission at a one of its higher harmonics. Such hybrid schemes will be the subject of future research.

The equations were also solved to demonstrate the same seeding mechanism for a fifth harmonic interaction with $n=5$ and for a current ratio $R_5=4$. A similar interaction to that for the $n=3$ case is seen from Fig. 5. As with the $n=3$ case, the value of the current ratio $R_5=4$ is below the threshold of Eq. (17), so that the fundamental of the lower energy beam will have a larger growth rate in the linear regime. The third harmonic field has no resonant coupling with the higher energy beam and it can be seen that the fifth harmonic has a greater linear growth rate ($0<\bar{z}<5$) and saturation intensity. The evolution of the wave equation driving terms S_φ and $S_{5\vartheta}$ are also similar to that for $n=3$.

IV. CONCLUSIONS

A simple 1D model of a two-electron beam FEL has been presented. This concept introduces potentially interesting coupled FEL interactions, some of which may have beneficial properties over single beam interactions. Indeed, one need not be limited to only two beams and can envisage more complex systems with more than two harmonically coupled beams—a multibeam FEL equivalent of a cascaded HGHG scheme [4]. Such multibeam FELs offer the prospect of a reduction in overall length when compared with the equivalent HGHG scheme. One can also envisage possible hybrid schemes involving combinations of multibeam FELs with HGHG. Clearly, these suggestions are purely speculative at this stage and will require further research.

As an illustration of the type of interactions possible, numerical simulations of the coupled two-beam FEL interaction demonstrated that a seeded lower energy beam interaction may evolve to effectively seed that of the higher energy. Seeding of both the radiation field *and* the electron bunching occurs. The conjecture was made that the improved coherence properties of the seeded interaction at the longer wavelength would be inherited by the higher beam energy, shorter wavelength, interaction. Such seeding may be of interest to proposals for FELs operating at sub-vuv wavelengths where no “conventional” seed sources are yet available. Further analysis is required to verify any predicted improvement in the coherence properties. Additional potential benefits of the two-beam FEL include the self-synchronized nature of the output at both the longer fundamental and shorter harmonic wavelength, which may prove a useful experimental source.

No attempt has been made to assess the importance of beam quality factors that will effect such two-beam interactions, e.g., accelerator physics issues such as electron pulse synchronism, beam emittance and energy spread, and the relative beam energy detuning between the electron beams which will clearly be important factors that will require further research. Other modes of FEL operation, such as the FEL oscillator and klystron-type configurations, would also be expected to have interesting properties when operated with two or more coupled resonant beams.

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